Cycle-Number Determination via the Hough Transform: 

The Technique and an Application to GW Cephei

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Received 1993 September 29; accepted 1994 March 23

ABSTRACT. Given a few determinations of the occurrence of a periodic event spanning a large interval of time, one can hope to make detailed determinations of the period and its behavior with time. This is accomplished by obtaining the correct cycle number for each observation: we show here that a modification of the Hough transform provides a robust technique for this detection. Very few observations of the event could then be all that is needed. An application of such a method to the eclipsing binary GW Cephei is given, showing that the most likely period change of this variable is opposite in sign and approximately one order of magnitude smaller than currently estimated.

1. STATEMENT OF THE PROBLEM

The determination of the period \( P \) and of period changes \( (dP/dt, d^2P/dt^2, \text{and so on}) \) from photometric studies of the light curves of variable objects such as eclipsing binaries, is simple in principle but meets in practice with several obstacles inherent to the utilization of telescope time, seldom under the sole control of one observer. This paper gives a method to overcome some such problems.

Let us examine the case of three observations taken at times \( t_1, t_2, t_3 \) (see Fig. 1), each time \( t_i \) being related to a well-defined cycle number \( E_i \); time and cycle differences will be denoted in the following by

\[ \Delta t_i = t_i - t_1, \quad \Delta E_i = E_i - E_1. \]  

(1)

More generally, one could consider three groups of observations, each obtained during an interval \( \Delta t_i \) around instants \( t_i \); if \( \Delta_i \leq \Delta t_i \) we obtain the same conditions of three single data.

Supposing the three observations exactly match a constant period \( P \), the following relationship holds:

\[ \eta = \frac{\Delta t_{12}}{\Delta t_{13}} = \frac{E_2 - E_1}{E_3 - E_2}. \]  

(2)

defining \( \eta \). If, in the last observation, a cycle-numbering error \( n \) occurs, a different best fit will be obtained, characterized by a false period \( P^* \neq P \) given by

\[ P^* = \frac{\Delta t_{13}}{\Delta E_{13} + n}. \]  

(3)

The resulting maximum error in the fit is given by

\[ \Delta \varphi = \frac{1}{2} \left( \frac{\Delta E_{12} + \Delta t_{12}}{P^*} \right) = \frac{1}{2} \left( \frac{m - n}{1 + \eta} \right), \]  

(4)

where \( m \) is the cycle-numbering error in the second observation and \( \Delta \varphi \) is given in period units, while the true-time error will be given by \( \Delta \tau = P \times \Delta \varphi \). Obviously, for any \( n \) a value \( m \) will exist that produces the lowest \( \Delta \varphi \).

In Fig. 2 the relationship (3) is plotted for various \( n \) and the corresponding \( m \) values that minimize the \( \Delta \varphi \) function. The values of \( m \) are reported on each segment of the plotted curves.

As one can see, assuming a threshold for the \( \Delta \varphi \) value compatible with the timing errors, a set of \( \eta \) ranges is obtained. If observations fall in times characterized by \( \eta \) values compatible with the obtained ranges, care has to be given to the possibility of encountering cycle-numbering problems.

For any \( n, n + 1 \) such intervals exist; furthermore, when \( n \) is odd a minimum around \( \eta = 1 \) is obtained, otherwise a minimum around \( \eta = 2 \) (and a corresponding minimum at \( \eta = 0.5 \)) is obtained. As shown below, large numbers for \( n \) are constrained by the accuracy attainable for each group of observations. The widths of the allowed \( \eta \) values are listed in Table 1, assuming an accuracy \( \Delta \varphi = 0.05 \).

It is worth noting that a false period \( P^* \) can be rejected a priori upon the knowledge given by each group of observations, where the cycle-numbering problem is absent, because of the very low number of elapsed cycles.

While the allowed variation \( \Delta P_{\text{max}} = P - P^* \) is given, approximately, by

\[ \Delta P_{\text{max}} = \frac{\Delta t_{13}}{(\Delta E_{13} + n)^2} \cdot \frac{n \cdot P}{\Delta E} \cdot n \]  

(5)

a single group of observations, assuming a maximum phase error \( \Delta \varphi = \Delta \tau \), will impose the constraint

\[ \Delta P_{\text{max}} \approx \frac{P \Delta \varphi}{\Delta E_i} \]  

(6)

for each \( i \)th observation group. Combining the last two equations and solving for \( n \), one obtains

\[ n_{\text{max}} \approx \frac{\Delta E_{13}}{\Delta E_i} \cdot \Delta \varphi. \]  

(7)
This value has to be regarded as an upper constraint in the allowed regions of Fig. 2 where cycle-numbering errors could be occurring.

2. A MODIFICATION OF THE HOUGH TRANSFORM

The Hough transform (and its various modifications, like the Fast version) was introduced (Hough, 1962) to solve the problem of the detection of patterns, especially for image-understanding problems.

Essentially it consists, for each pixel of the incoming image, in the determination of the specific parameters of the pattern to be detected. In the parameter space all the obtained values are accumulated and the pattern detection problem becomes a simpler peak detection.

A detailed description of the technique, of the possible applications, and of its statistic robustness, can be found in Illingworth and Kittler (1988) and Risse (1989); an astronomical application is given by Ballester (1991). A recent and more comprehensive book on the subject is the one by Leavers (1992).

In our case the problem of the identification of the ephemeris of the periodic event consists in the determination of the

Fig. 1—A cycle-numbering error of \( n \) will produce a false period determination \( P^* \) and a maximum error in the corresponding fit of \( \Delta \phi \).

Fig. 2—The maximum error \( \Delta \phi \) vs. \( \eta \) for various cycle-numbering errors \( n \).

Fig. 3—Generalizing Hough Transform (HT) of a single minimum observed in \( t = t^* \) when no \textit{a priori} information about \( E^* \) is assumed.

The Allowed Ranges for the Ratio \( \eta \) Compatible with a Cycle-Numbering Error of \( n \) Assuming a Maximum Error \( \Delta \phi = 0.05 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \eta = 0.5 )</th>
<th>( \eta = 1 )</th>
<th>( \eta = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.8 to 1.25</td>
<td>0.8 to 1.25</td>
<td>0.8 to 1.25</td>
</tr>
<tr>
<td>3</td>
<td>0.4 to 0.6</td>
<td>0.6 to 1.15</td>
<td>0.9 to 1.15</td>
</tr>
<tr>
<td>4</td>
<td>0.6 to 0.9</td>
<td>0.9 to 1.15</td>
<td>1.7 to 2.35</td>
</tr>
</tbody>
</table>

The straight line closest to the observed minimum (see Fig. 3). The slope of the line is the parameter identifiable with \( P \), while the intercept \( t_0 \) of the line with the axes \( t \) will identify the epoch or, equally, the phase \( \phi_0 \) at an arbitrary given epoch \( t \).

The usual Hough transform will associate to each point \( P_0 \) in the \( E, t \) plane (that is, to each observed minima at \( t^* \) associated with a well-defined cycle number \( E^* \)) a curve \( C_0 \) in the parameter space \( P, \phi_0 \) given by

\[
\phi_0 = \frac{t^* - t}{P} E^*. \tag{8}
\]

This operation is called back projection. The intersection of the whole set of back projections given by the whole set of observations will yield the identification of the correct line to be used (that is, the ephemeris of the variable) only if the proper choice of \( E^* \)'s was made, that is, if no cycle-numbering error occurred.

The modification proposed in this paper will free the choice in \( E^* \) allowing the observations to refer to the full set of points \( P_{-\infty} \) to \( P_{+\infty} \) producing in the parameter space a set of curves \( C_{-\infty} \) to \( C_{+\infty} \).

In practice a small box in the \( (P, \phi_0) \) plane is selected and for each observation of a minimum falling at time \( t_i \), the bidimensional function

\[
\Delta \phi = \phi_0 - \frac{t_i - t}{P} E^*. \tag{9}
\]

is evaluated, \( \{k\} \) being the fractional part of \( k \). This function gives, for each \( i \)th observation and any pair \( P, \phi_0 \) in the as-
CYCLE NUMBERING VIA HOUGH TRANSFORM

Fig. 4—The Hough transform for the published minimum times of GW Cep.

sumed domain, the absolute phase difference between the ephemeris and the observation.

\[ \Delta \phi \] lies in the range \([0,1]\) but the departure from the correct ephemeris is given by the departure of \(\Delta \phi\) from the nearest integer number (in our case, between 0 and 1 only).

The function to be accumulated will be any function \(G(\Delta \phi)\) symmetric around 1/2 [that is, \(G(x) = G(1-x)\)] and with monotonically decreasing behavior between 0 and 1/2.

Suitable functions are, for instance,

\[ G(x) = \left| x - \frac{1}{2} \right| \]

or

\[ G(x) = x^2 - x. \]

At the end of this process a search for the maximum will give the best line fit, allowing to get the proper cycle numbering without constraints to a priori information. Being that \(O[\Delta \phi(P, \phi_0)]\) is evaluated in a discrete array of \((P, \phi_0)\) it is important to have a good choice of the step to be used. A detailed discussion about this point is given, for example, in Van Veen and Groen (1981). The overall performances of the method have been investigated in some detail through Monte Carlo simulations (Ragazzoni and Barbieri 1993).

3. AN APPLICATION CASE: GW CEP

GW Cep (=CSV 5941 = VB 7 Cep; \(a=0^h40^m41^s, \delta = +79^\circ49'9'', \) equinox 1950.0) was discovered as a variable by Geyer et al. (1955). The nature of the variability was investigated with some accuracy essentially by Hoffman (1982) and Kaluzny (1984), recognizing it as a W Ursae Majoris eclipsing variable with a total eclipse minimum. The primary minimum is constant and is some 0.05 mag deeper than the secondary one. This should allow to reject the possibility to have a misdetection of the true nature of the observed minimum. Details can be found in the cited references.

Regarding the prediction of minimum times, the determination of the ephemeris elements were given by Meinunger and Wenzel (1965) \((P = 0.31885)\), Hoffman (1982) \((P = 0.31884945)\), and Landolt (1992) \((P = 0.318851065)\). In this last paper the possibility to have problems in cycle counts was explicitly pointed out. The observations of GW Cep available in the literature consist of six minimum time determinations spanning a little less than 28 years; nevertheless they can be grouped into three epochs lying approximately at JD=2438537, JD=2444200, and JD=2448545. These observations define two time intervals of 5663 and 4345 days, forming a ratio of \(\eta^2\). As shown earlier [Eq. (3)], this will translate, when a two cycle-numbering error

### Table 2

<table>
<thead>
<tr>
<th>JD</th>
<th>(A)</th>
<th>(O-C)</th>
<th>(B)</th>
<th>(O-C)</th>
<th>(C)</th>
<th>(O-C)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2438383.711</td>
<td>-840</td>
<td>0.00089</td>
<td>-840</td>
<td>-0.02111</td>
<td>-840</td>
<td>-0.03181</td>
<td>a</td>
</tr>
<tr>
<td>2438462.474</td>
<td>-593</td>
<td>0.00768</td>
<td>-593</td>
<td>-0.00932</td>
<td>-593</td>
<td>-0.01503</td>
<td>a</td>
</tr>
<tr>
<td>2438651.545</td>
<td>0</td>
<td>0.00000</td>
<td>0</td>
<td>-0.00500</td>
<td>0</td>
<td>0.00130</td>
<td>a</td>
</tr>
<tr>
<td>2438652.503</td>
<td>3</td>
<td>0.00145</td>
<td>3</td>
<td>-0.00349</td>
<td>3</td>
<td>0.00287</td>
<td>a</td>
</tr>
<tr>
<td>2444200.482</td>
<td>17403</td>
<td>-0.02863</td>
<td>17404</td>
<td>0.00046</td>
<td>17405</td>
<td>0.04020</td>
<td>b</td>
</tr>
<tr>
<td>2448544.871</td>
<td>31028</td>
<td>0.01519</td>
<td>31030</td>
<td>0.00068</td>
<td>31032</td>
<td>-0.00259</td>
<td>c</td>
</tr>
</tbody>
</table>

Epoch [JD]: 2438537.5450 2438651.5500 2438651.5437

Period [days]: 0.318851065 0.31883082 0.31881058

\(dP/dt\): 1.528 \times 10^{-3} -2.327 \times 10^{-3} -1.993 \times 10^{-3}
(n = 2) occurs, into a phase lag $\Delta \varphi \approx 0.06$ corresponding to a lag $\Delta \tau \approx 0.02$ days.

This last value is compatible with the published $O-C$ values given by Landolt (1992), reinforcing on a numerical basis the suspicion of cycle-numbering problems.

The realization of the Hough transform for the published set of minimum times gives the result shown in Fig. 4, where the solution given by Landolt (1992), marked with A can be easily recognized; then a stronger peak B and a third solution C, with approximately the same height as A, are found.

The solutions given by Meinunger and Wenzel (1965) and Hoffman (1982) lie approximately in peak A (in a sense, Landolt’s value is a refinement of the two preceding solutions).

While it is clear that assuming a constant period the best solution (in the sense of lowest $O-C$ rms values) is given by the peak B, it is also clear that allowing for a period variation $dP/dt$ different from zero, there is no a priori reason to choose among A, B, or C. The three cases are shown in the panels of Fig. 5, where a parabolic fit ($dP/dt=$ const) is superimposed.

The resulting cycle numbers, $O-C$ values, and $dP/dT$ estimations are shown in Table 2.

As it can be easily seen, the three solutions, while differing by only $2 \times 10^{-7}$ days with respect to each other regarding the value of $P$, substantially disagree in the $dP/dT$ determination. Solution B gives a $dP/dT$ one order of magnitude smaller than A and C, the latter giving two values with opposite signs.

It is worth noting that an extremely accurate determination of $P$ is almost meaningless from the physical point of view, while the determination of period changes, with its implication on the mass-flow problems in contact binary stars is a notoriously unresolved problem, essentially due to lack of data on the subject. See, for example, Eggleton et al. (1991) and Pringle and Wade (1985).

It is important to stress that the obtained values for $dP/dt$ are only an indication of what could be the behavior of the star, being that the true $P$ story of a W UMa star is generally not described by the simple assumption of a constant $dP/dt$, see, for example, Yamasaki (1975).

In order to gain further insight in the real behavior of GW Cep, we have searched the archive of plates of the Asiago S67 Schmidt telescope, available under computer environment (Franceschi 1989). Three Super Ortho-Press plates were found, taken with double exposure with a slight offset of the telescope pointing, in order to detect flare stars. Care was given to exclude the possibility of overlapping of the image of the GW Cep with nearby stars. The log of the plates is reported in Table 3.

Plates were digitized with the Padova Observatory PDS machine and photometry was performed using DAOphot (Stetson 1987) and a crude aperture photometry. For photometric reference, the stars designed C and S (see Landolt 1992) were used. No attempt for color photometric correction was made. It is not possible to directly compare the measured magnitudes with the expected ones. On the other hand it is always possible, being that the color index of GW Cep is nearly constant over the whole cycle, to compare differences of magnitudes in different epochs with the expected ones.

Table 4 reports the expected phases for GW Cep depending upon the various (A, B, and C) hypotheses, and the corresponding magnitudes obtained from the curve published by Hoffman (1982).

The inferred differences in magnitude are significantly different for the three cases (up to 1 mag, between the A and B options) and are plotted in Fig. 6 with the measured values. The error bars in the graph refer to the dispersion obtained from the different reduction algorithm, from a different choice of the reference star (only C, only S, or both) and comparing the two images of the star found in each plate.

The agreement between the measured values and the B option is quite good, giving the most likely period and period variation in the values of Table 2, column B.

4. CONCLUSIONS

This paper outlines the sensitivity of observational data to the cycle-numbering problem, examining in detail the case of
three groups of observations. Indications for the extension to more than three groups are briefly given. A modification of the Hough transform, to deal with this type of problem, is explained in detail. Furthermore, indications are presented for the time windows to be chosen in order to experimentally solve the debate about cycle numbering. An application to the eclipsing binary star GW Cep is made, demonstrating the efficacy of the method and giving a new determination for the period \( P \) and the possible period variation \( dP/dt \).

The obtained value for \( dP/dt \) agrees in sign and order of magnitude with the ones obtained for AW UMa (Hrivnak 1982) and V1010 Ophiuchi (Dreschel et al. 1977), however it remains of the opposite sign and largely higher than the value predicted by the current scenarios for this type of star (Vilhu 1991, assumes a \( dP/dt = 10^{-7} \) \( P/\)year\(^{-1} \times 10^{-12} \) for GW Cep).

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